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Generalization of Ramanujan's identities in terms of q-products and continued fractions

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GENERALIZATIONOF RAMANUJANS IDENTITIESIN TERMS OF Q-PRODUCTS AND CONTINUEDFRACTIONS

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3. B.C. Berndt; *Ramanujan's notebook Part III*, Springer-Verlag, New York, 1991.

Generalization of Ramanujan's identities in terms of q-products and continued fractions

M.P. Chaudhary

Abstract - In this paper, we generalized seven Ramanujan's identities in terms of q-products and continued fractions, using properties of Jacobi's triple product identities. Findings are new and not available in the literature of special functions.

Keywords : *Jacobi's triple product identities, q-products and continued fraction.*

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1-aq^{(n-1)}) \tag{1.2}$$

$$(a_1,a_2,a_3,...,a_k;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan [2, p.1(1.2)] has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [3,p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\phi(q) = f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)^2_\infty(q^2;q^2)_\infty \tag{1.6}$$

$$(q) = f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

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$$f(-q) = f(-q, -q^2) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty (q^b;q^n)_\infty (q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^b,q^c \cdots q^t;q^n)_\infty \tag{1.10}$$

$$(q^a;q^n)_\infty (q^b;q^n)_\infty (q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^b,q^c \cdots q^t;q^n)_\infty \tag{1.11}$$

$$(-q^a;q^n)_\infty (-q^b;q^n)_\infty (q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (-q^a,-q^b,q^c \cdots q^t;q^n)_\infty \tag{1.12}$$

Computation of q-product identities:

Now we can have following q-products identities, as

$$(q^2;q^2)_\infty = \prod_{n=0}^{\infty}(1-q^{2n+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{2(4n)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+1)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+2)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{8n+2}) \times \prod_{n=0}^{\infty}(1-q^{8n+4}) \times \prod_{n=0}^{\infty}(1-q^{8n+6}) \times \prod_{n=0}^{\infty}(1-q^{8n+8})$$

or,

$$\begin{aligned}(q^2;q^2)_\infty &= (q^2;q^8)_\infty (q^4;q^8)_\infty (q^6;q^8)_\infty (q^8;q^8)_\infty \\ &= (q^2,q^4,q^6,q^8;q^8)_\infty\end{aligned} \tag{1.13}$$

also we can compute

$$(q^2;q^2)_\infty = (q^2;q^4)_\infty (q^4;q^4)_\infty \tag{1.14}$$

$$(q^4;q^4)_\infty = \prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{4(3n)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+1)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{12n+4}) \times \prod_{n=0}^{\infty}(1-q^{12n+8}) \times \prod_{n=0}^{\infty}(1-q^{12n+12})$$

or,

$$\begin{aligned}(q^4;q^4)_\infty &= (q^4;q^{12})_\infty (q^8;q^{12})_\infty (q^{12};q^{12})_\infty \\ &= (q^4,q^8,q^{12};q^{12})_\infty\end{aligned} \tag{1.15}$$

$$(q^4;q^{12})_\infty = \prod_{n=0}^{\infty}(1-q^{12n+4}) = \prod_{n=0}^{\infty}(1-q^{12(5n)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+1)+4}) \times$$

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+2)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+3)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

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$$= \prod_{n=0}^{\infty}(1-q^{60n+4}) \times \prod_{n=0}^{\infty}(1-q^{60n+16}) \times \prod_{n=0}^{\infty}(1-q^{60n+28}) \times$$
$$\times \prod_{n=0}^{\infty}(1-q^{60n+40}) \times \prod_{n=0}^{\infty}(1-q^{60n+52})$$

or,

$$(q^4;q^{12})_\infty = (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$
$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \quad (1.16)$$

Similarly we can compute following as

$$(q^5;q^5)_\infty = (q^5;q^{15})_\infty(q^{10};q^{15})_\infty(q^{15};q^{15})_\infty$$
$$= (q^5,q^{10},q^{15};q^{15})_\infty \quad (1.17)$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty(q^{12};q^{24})_\infty(q^{18};q^{24})_\infty(q^{24};q^{24})_\infty$$
$$= (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \quad (1.18)$$

$$(q^6;q^{12})_\infty = (q^6;q^{60})_\infty(q^{18};q^{60})_\infty(q^{30};q^{60})_\infty(q^{42};q^{60})_\infty(q^{54};q^{60})_\infty$$
$$= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty \quad (1.19)$$

The outline of this paper is as follows. In sections 2, some results on continued fraction [5-8], and also some well known results recorded by Ramanujan [9], are listed, those are useful to the rest of the paper. In section 3, we established seven new results by generalizing Rmanujan's identities in terms of q-products and continued fractions, using the properties Jacobi's triple product identities. Findings are new and not available in the literature of special functions. In section 4, we provide the proofs for newly established results.

II. Preliminaries

In [9, p. 224], Ramanujan recorded following identities

Entry(i):

$$\frac{(q^7)\ \ (q^9) - \ \ (-q^7)\ \ (-q^9)}{(q)\ \ (q^{63}) - \ \ (-q)\ \ (-q^{63})} = q^6 \quad (2.1)$$

Entry(ii):

$$\frac{(q^5)\ \ (q^{11}) - \ \ (-q^5)\ \ (-q^{11})}{(q)\ \ (q^{55}) - \ \ (-q)\ \ (-q^{55})} = q^5 \quad (2.2)$$

Entry(iii):

$$\frac{(q^3)\ \ (q^{13}) - \ \ (-q^3)\ \ (-q^{13})}{(q)\ \ (q^{39}) - \ \ (-q)\ \ (-q^{39})} = q^3 \quad (2.3)$$

In [9, p. 230], Ramanujan recorded following identities

Entry(vii):

$$(q)\ \ (q^{11}) - \ \ (-q)\ \ (-q^{11}) = 2qf(q^2,q^{10})f(q^{44},q^{88}) + 2q^{15}\phi(q^6)\ \ (q^{132}) \quad (2.4)$$

Ref.

5. G.E. Andrews; *An introduction to Ramanujan's Lost notebook*, Amer. Math. Monthly. 86(2)(1979), 89-98.
9. S. Ramanujan; *Notebooks (Volume I)*, Tata Institute of Fundamental Research, Bombay, 1957.

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In [9, p. 299], Ramanujan recorded following identities

Entry(ii):

$$\phi(q)\phi(q^{27}) - \phi(-q)\phi(-q^{27}) = 4qf(-q^6)f(-q^{18}) + 4q^7\ \ (q^2)\ \ (q^{54}) \tag{2.5}$$

Entry(iii):

$$\phi(q)\phi(q^{35}) - \phi(-q)\phi(-q^{35}) = 4qf(-q^{10})f(-q^{14}) + 4q^9\ \ (q^2)\ \ (q^{70}) \tag{2.6}$$

Entry(iv):

$$\phi(q^5)\phi(q^7) - \phi(-q^5)\phi(-q^7) = 4q^3\ \ (q^{10})\ \ (q^{14}) - 2q^3f(-q^2)f(-q^{70}) \tag{2.7}$$

In [7], following continued fractional identities is given

$$(q^2;q^2)_\infty(-q;q)_\infty = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} = \cfrac{1}{1 - \cfrac{q}{1 + \cfrac{q(1-q)}{1 - \cfrac{q^3}{1 + \cfrac{q^2(1-q^2)}{1 - \cfrac{q^5}{1 + \cfrac{q^3(1-q^3)}{1 + \vdots}}}}}}} \tag{2.8}$$

Following Rogers-Ramanujan continued fraction is one of the most celebrated identities associated with Ramanujan's academic career [8],

$$C(q) = \frac{(q^2;q^5)_\infty(q^3;q^5)_\infty}{(q;q^5)_\infty(q^4;q^5)_\infty} = 1 + \cfrac{q}{1 + \cfrac{q^2}{1 + \cfrac{q^3}{1 + \cfrac{q^4}{1 + \cfrac{q^5}{1 + \vdots}}}}} \tag{2.9}$$

In [5, equation (1.6)], the famous Rogers-Ramanujan continued fraction identity is given

$$\frac{(q;q^5)_\infty(q^4;q^5)_\infty}{(q^2;q^5)_\infty(q^3;q^5)_\infty} = \cfrac{1}{1 + \cfrac{q}{1 + \cfrac{q^2}{1 + \cfrac{q^3}{1 + \cfrac{q^4}{1 + \vdots}}}}} \tag{2.10}$$

In [6, equation (4.21)], following Ramanujan continued fraction identity is given

$$\frac{(-q^3;q^4)_\infty}{(-q;q^4)_\infty} = \cfrac{1}{1 + \cfrac{q}{1 + \cfrac{q^3+q^2}{1 + \cfrac{q^5}{1 + \cfrac{q^7+q^4}{1 + \cfrac{q^9}{1 + \cfrac{q^{11}+q^6}{1 + \vdots}}}}}}} \tag{2.11}$$

Ref.

7. R.Y. Denis; *On certain q-series and continued fractions*, Math. Students, 44(1-4)(1983), 70-76.
9. S. Ramanujan; *Notebooks (Volume I)*, Tata Institute of Fundamental Research, Bombay, 1957.

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III. Main Results

In this section, we established seven new results by using (.) and $\phi(.)$ functions in Ramanujan identities [9], or in more general language we can say that by using the properties of Jacobi's triple product identity, as (.) and $\phi(.)$ functions are special cases of it, and further applying the properties of continued fraction identities. These results are new, and not recorded in the literature of special functions

$$q^6 = \left[\frac{(-q^7;q^{14})_\infty(-q^9;q^{18})_\infty - (q^7;q^{14})_\infty(q^9;q^{18})_\infty}{(-q;q^2)_\infty(-q^{63};q^{126})_\infty - (q;q^2)_\infty(q^{63};q^{126})_\infty}\right]\times$$

$$\times\frac{(-q,q;q^2)_\infty(-q^{63},q^{63};q^{126})_\infty}{(q^2;q^2)_\infty(-q^7;q^{14})_\infty(-q^9;q^{18})_\infty(q^{126};q^{126})_\infty}\times$$

$$\times\cfrac{1}{1-\cfrac{q^7}{1+\cfrac{q^7(1-q^7)}{1-\cfrac{q^{21}}{1+\cfrac{q^{14}(1-q^{14})}{1-\cfrac{q^{35}}{1+\cfrac{q^{21}(1-q^{21})}{1+\vdots}}}}}}}\times\cfrac{1}{1-\cfrac{q^9}{1+\cfrac{q^9(1-q^9)}{1-\cfrac{q^{27}}{1+\cfrac{q^{18}(1-q^{18})}{1-\cfrac{q^{45}}{1+\cfrac{q^{27}(1-q^{27})}{1+\vdots}}}}}}} \quad (3.1)$$

$$q^5 = \left[\frac{(-q^5;q^{10})_\infty(-q^{11};q^{22})_\infty - (q^5;q^{10})_\infty(q^{11};q^{22})_\infty}{(-q;q^2)_\infty(-q^{55};q^{110})_\infty - (q;q^2)_\infty(q^{55};q^{110})_\infty}\right]\times$$

$$\times\frac{(-q,q;q^2)_\infty(-q^{55},q^{55};q^{110})_\infty}{(q^2;q^2)_\infty(-q^5;q^{10})_\infty(-q^{11};q^{22})_\infty(q^{110};q^{110})_\infty}\times$$

$$\times\cfrac{1}{1-\cfrac{q^5}{1+\cfrac{q^5(1-q^5)}{1-\cfrac{q^{15}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{25}}{1+\cfrac{q^{15}(1-q^{15})}{1+\vdots}}}}}}}\times\cfrac{1}{1-\cfrac{q^{11}}{1+\cfrac{q^{11}(1-q^{11})}{1-\cfrac{q^{33}}{1+\cfrac{q^{22}(1-q^{22})}{1-\cfrac{q^{55}}{1+\cfrac{q^{33}(1-q^{33})}{1+\vdots}}}}}}} \quad (3.2)$$

$$q^3 = \left[\frac{(-q^3;q^6)_\infty(-q^{13};q^{26})_\infty - (q^3;q^6)_\infty(q^{13};q^{26})_\infty}{(-q;q^2)_\infty(-q^{39};q^{78})_\infty - (q;q^2)_\infty(q^{39};q^{78})_\infty}\right]\times$$

$$\times\frac{(-q,q;q^2)_\infty(-q^{39},q^{39};q^{78})_\infty}{(q^2;q^2)_\infty(q^{78};q^{78})_\infty(-q^3;q^6)_\infty(-q^{13};q^{26})_\infty}\times$$

Ref.

9. S. Ramanujan; *Notebooks (Volume I)*, Tata Institute of Fundamental Research, Bombay, 1957.

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$$\times \cfrac{1}{1-\cfrac{q^3}{1+\cfrac{q^3(1-q^3)}{1-\cfrac{q^9}{1+\cfrac{q^6(1-q^6)}{1-\cfrac{q^{15}}{1+\cfrac{q^9(1-q^9)}{1+\vdots}}}}}}} \times \cfrac{1}{1-\cfrac{q^{13}}{1+\cfrac{q^{13}(1-q^{13})}{1-\cfrac{q^{39}}{1+\cfrac{q^{26}(1-q^{26})}{1-\cfrac{q^{65}}{1+\cfrac{q^{39}(1-q^{39})}{1+\vdots}}}}}}} \tag{3.3}$$

$$2q(q^{12};q^{12})_\infty\left[(-q^2,-q^{10};q^{12})_\infty(-q^{44},q^{88};q^{132})_\infty+q^{14}(-q^6;q^{12})^2_\infty\frac{(q^{264};q^{264})_\infty}{(q^{132};q^{264})_\infty}\right]$$

$$=\left[\frac{(-q;q^2)_\infty(-q^{11};q^{22})_\infty-(q;q^2)_\infty(q^{11};q^{22})_\infty}{(-q;q^2)_\infty(-q^{11};q^{22})_\infty}\right]\times$$

$$\times \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \times \cfrac{1}{1-\cfrac{q^{11}}{1+\cfrac{q^{11}(1-q^{11})}{1-\cfrac{q^{33}}{1+\cfrac{q^{22}(1-q^{22})}{1-\cfrac{q^{55}}{1+\cfrac{q^{33}(1-q^{33})}{1+\vdots}}}}}}} \tag{3.4}$$

$$(q^2;q^2)_\infty(q^{54};q^{54})_\infty\left[(-q;q^2)^2_\infty(-q^{27};q^{54})^2_\infty-(q;q^2)^2_\infty(q^{27};q^{54})^2_\infty\right]$$

$$=4q(q^6;q^6)_\infty(q^{18};q^{18})_\infty+4q^7\times$$

$$\times \cfrac{1}{1-\cfrac{q^2}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^6}{1+\cfrac{q^4(1-q^4)}{1-\cfrac{q^{10}}{1+\cfrac{q^6(1-q^6)}{1+\vdots}}}}}}} \times \cfrac{1}{1-\cfrac{q^{54}}{1+\cfrac{q^{54}(1-q^{54})}{1-\cfrac{q^{162}}{1+\cfrac{q^{108}(1-q^{108})}{1-\cfrac{q^{270}}{1+\cfrac{q^{162}(1-q^{162})}{1+\vdots}}}}}}} \tag{3.5}$$

$$(q^2;q^2)_\infty(q^{70};q^{70})_\infty\left[(-q;q^2)^2_\infty(-q^{35};q^{70})^2_\infty-(q;q^2)^2_\infty(q^{35};q^{70})^2_\infty\right]$$

$$=4q(q^{10};q^{10})_\infty(q^{14};q^{14})_\infty+4q^9\times$$

$$\times \cfrac{1}{1-\cfrac{q^2}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^6}{1+\cfrac{q^4(1-q^4)}{1-\cfrac{q^{10}}{1+\cfrac{q^6(1-q^6)}{1+\vdots}}}}}}} \times \cfrac{1}{1-\cfrac{q^{70}}{1+\cfrac{q^{70}(1-q^{70})}{1-\cfrac{q^{210}}{1+\cfrac{q^{140}(1-q^{140})}{1-\cfrac{q^{350}}{1+\cfrac{q^{210}(1-q^{210})}{1+\vdots}}}}}}} \tag{3.6}$$

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$$(q^{10};q^{10})_\infty(q^{14};q^{14})_\infty\left[(-q^5;q^{10})^2_\infty(-q^7;q^{14})^2_\infty-(q^5;q^{10})^2_\infty(q^7;q^{14})^2_\infty\right]$$

$$=-2q^3(q^2;q^2)_\infty(q^{70};q^{70})_\infty+4q^3\times$$

$$\times\cfrac{1}{1-\cfrac{q^{10}}{1+\cfrac{q^{10}(1-q^{10})}{1-\cfrac{q^{30}}{1+\cfrac{q^{20}(1-q^{20})}{1-\cfrac{q^{50}}{1+\cfrac{q^{30}(1-q^{30})}{1+\vdots}}}}}}}\times\cfrac{1}{1-\cfrac{q^{14}}{1+\cfrac{q^{14}(1-q^{14})}{1-\cfrac{q^{42}}{1+\cfrac{q^{28}(1-q^{28})}{1-\cfrac{q^{70}}{1+\cfrac{q^{42}(1-q^{42})}{1+\vdots}}}}}}} \quad (3.7)$$

IV. Proofs For Main Results (3.1) to (3.7)

Proof of (3.1): In (1.7), put $q=-q,q^7,-q^7,q^9,-q^9,q^{63},-q^{63}$ respectively, we get

$$(-q)=\frac{(q^2;q^2)_\infty}{(-q;q^2)_\infty},\ \psi(q^7)=\frac{(q^{14};q^{14})_\infty}{(q^7;q^{14})_\infty},\ \psi(-q^7)=\frac{(q^{14};q^{14})_\infty}{(-q^7;q^{14})_\infty} \quad (3.1.1)$$

$$(q^9)=\frac{(q^{18};q^{18})_\infty}{(q^9;q^{18})_\infty},\ \psi(-q^9)=\frac{(q^{18};q^{18})_\infty}{(-q^9;q^{18})_\infty} \quad (3.1.2)$$

$$(q^{63})=\frac{(q^{126};q^{126})_\infty}{(q^{63};q^{126})_\infty},\ \psi(-q^{63})=\frac{(q^{126};q^{126})_\infty}{(-q^{63};q^{126})_\infty} \quad (3.1.3)$$

Now, substituting the values from (3.1.1) to (3.1.3), and using (1.7) into (2.1), after simplifications by applying the properties of q-product identities and further using continued fraction (2.8), we get desired result (3.1).

Proofs of (3.2) and (3.3): On similar lines of proof for (3.1), we can easily obtain proofs for (3.2) and (3.3).

Proof of (3.4): In (1.7), put $q=-q,q^{11},-q^{11},q^{132}$, respectively, we get

$$(-q)=\frac{(q^2;q^2)_\infty}{(-q;q^2)_\infty},\ \psi(q^{11})=\frac{(q^{22};q^{22})_\infty}{(q^{11};q^{22})_\infty},\ \psi(-q^{11})=\frac{(q^{22};q^{22})_\infty}{(-q^{11};q^{22})_\infty},\ \psi(q^{132})=\frac{(q^{264};q^{264})_\infty}{(q^{132};q^{264})_\infty} \quad (3.4.1)$$

again by putting $q=q^6$ in (1.6), we get

$$\phi(q^6)=(-q^6;q^{12})^2_\infty(q^{12};q^{12})_\infty \quad (3.4.2)$$

also by putting $a=q^2,b=q^{10}$ and $a=q^{44},b=q^{88}$ respectively in (1.5), we get

$$f(q^2,q^{10})=(-q^2;q^{12})_\infty(-q^{10};q^{12})_\infty(q^{12};q^{12})_\infty \quad (3.4.3)$$

$$f(q^{44},q^{88})=(-q^{44};q^{132})_\infty(-q^{88};q^{132})_\infty(q^{132};q^{132})_\infty \quad (3.4.4)$$

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Now, substituting the values from (3.4.1) to (3.4.4), and using (1.7) into (2.4), after simplifications by applying the properties of q-product identities and further using continued fraction (2.8), we get desired result (3.4).

Proof of (3.5): In (1.6), put $q = -q, q^{27}, -q^{27}$, respectively, we get

$$\phi(-q) = (q;q^2)^2_\infty (q^2;q^2)_\infty \tag{3.5.1}$$

and

$$\phi(q^{27}) = (-q^{27};q^{54})^2_\infty (q^{54};q^{54})_\infty,\ \phi(-q^{27}) = (q^{27};q^{54})^2_\infty (q^{54};q^{54})_\infty \tag{3.5.2}$$

by substituting $q = q^2,\ q^{54}$ respectively in (1.7), we get

$$(q^2) = \frac{(q^4;q^4)_\infty}{(q^2;q^4)_\infty},\ \psi(q^{54}) = \frac{(q^{108};q^{108})_\infty}{(q^{54};q^{108})_\infty} \tag{3.5.3}$$

again by substituting $q = q^6,\ q^{18}$ respectively in (1.8), we get

$$f(-q^6) = (q^6;q^6)_\infty,\ f(-q^{18}) = (q^{18};q^{18})_\infty \tag{3.5.4}$$

Now, substituting the values from (3.5.1) to (3.5.4), and using (1.6) into (2.5), after simplifications by applying the properties of q-product identities and further using continued fraction (2.8), we get desired result (3.5).

Proofs of (3.6) and (3.7): On similar lines of proof for (3.5), we can easily obtain proofs for (3.6) and (3.7).

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